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Investigation of a 100-Foot Concrete Arch

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INVESTIGATION

OF A

100-FOOT CONCRETE ARCH

BY

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This is to certify that the thesis prepared under the immediate supervision of Instructor L. A. Waterbury by

LAWRENCE SWASEY KEELER

entitled INVESTIGATION OF A 100-FOOT CONCRETE ARCH

is approved by me as fulfilling this part of the requirements for the degree of Bachelor of Science in Civil Engineering

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Head of Department of Civil Engineering

INVESTIGATION OF A 100-FOOT CONCRETE ARCH

INTODUCTION.

The object of this thesis is to investigate stresses in the 100-foot concrete arch in the west approach of the Thebes Bridge, at Thebes, Illinois, by the method given in Cain's Steel-Concrete Arches. The arch is circular in form, but of variable section. The intrados of the arch ring has a radius of 50 feet, and the extrados a radius of 62 feet 1 5/8 inches, to a point 40° 1' from the vertical, from which point it follows the tangent line. The radical depth of the arch ring is 4 feet 6 inches at the crown, and 11 feet at the haunches. The arch is covered with earth filling, the depth of which is 4 feet 5 inches above the crown. Between the top of the earth filling and the bottom of the ties there is 1 foot of ballast. Retaining walls extend 12 feet above the intrados of the arch at the crown, flush with the faces of the arch ring. The thickness of these walls varies from 3 feet at the top to 12 feet at the bottom.

The arch is constructed of plain concrete, but several light iron rods were inserted to keep pieces of concrete from falling out in case of severe cracks. These rods are 1 1/4 inches square, two of which are 1 foot from the extrados, and 1 and 2 feet respectively from the face, on each side of the arch.

INVESTIGATION.

For this investigation a portion of the arch ring 1 foot wide taken near the center of the arch, was used. The train load was assumed as 6200 lb. per foot of track, extending from the left a-

butment to the center of the arch. The earth and concrete were assumed to weigh 100 and 150 lb. per cubic foot, respectively. Also, the track itself was estimated to weigh 350 lb. per lineal foot. The combined track and train loads were considered to be distributed, by the earth filling, over a width of 12 feet of the arch ring.

Having drawn the arch ring as shown on page 8, the neutral axis was next drawn and divided into 1 foot divisions. At these 1 foot points the depth of the arch ring was scaled, being, in each case, measured perpendicular to the neutral axis. These values are given in Table 1, page 9.

The lengths of the arch divisions were next determined, commencing at the crown and extending to the haunches, from the relation $\frac{s_1}{d_1^3} = \frac{s_2}{d_2^3}$, s being the length of any division, and d the radial dept[illegible] the section atthe same point. The values of s are given in Table II, page 10, in which l is the distance from the crown. As the arch is symetrical the divisions were laid off from the crown, both to the right and to the left, and were numbered from the right abutment to the left, s_1, s_2, s_3, ---.

Vertical lines were then drawn through the middle points of s_1, s_2, s_3,---, and designated by a_1, a_2, a_3, ---. The total load between the consecutive verticals were found by laying off the live and dead loads to a scale of their relative weights, and scaling the ordinates for the result, which are denoted by P_1, P_2, P_3, ---, and are applied midway between the points a_1, a_2, a_3,---. The values of P are given in Table III, page 11.

The successive loads P_1, P_2, P_3,---P_{32}, were laid off on a

vertical line and a trial thrust H, of 75000 lb. was assumed. The trial pole was then located by laying off the thrust to the scale of the loads from the line to a point horizontally to the right of the bottom of P_{16}. The trial equilibrium pologon was drawn in the usual manner, and the ordinates v, b, --- v_{32}, b_{32}, measured to the scale of the arch. Also the distances from v_1, v_2, v_3, --, to AB were scaled, and designated by z_1, z_2, z_3, ---. Moments (v b. z) were then taken about AB and the amount and position of the resultant R was found. The values of vb and z are given in Table IV, page 11.

A line mm, was then determined, such that if the ordinates from mm_1, to b_1, b_2, b_3, ---, were treated as forces, their resultant would coincide with R in amount and position. To this end the line nn_1 was assumed, and also a line from n to b_1, which divided the ordinates between v_1--v_{32} and nn_1, into two sets, viz:those in the triangle nn_1v_1 and those in triangle n v_1 v_{32}. For convenience, in drawing n n_1, v_{32} n_{32} and v_1n_1 were taken as 15 units, so that the two triangles mentioned were equal and had equal ordinates. The designation "trial T" was given to the ordinates, considered as forces, in the triangle n n_1 v_1, and "trial T′" to the ordinates in n v_1 v_{32}. Taking moments about AB the position and amount of "trial T" were ascertained. It is obvious that trial T equals T′, and that they act at equal distances from AB. The values of vn are given in Table V, page 13.

To find the true value of T and T′ moments were taken about T and T′ in turn.

$$\text{true } T = \frac{(526.63)(12.053)}{22.968} = 275.14$$

$$\text{true } T' = \frac{(526.63)(10.915)}{22.968} = 250.27$$

To locate mm_1 ,

$$v_{32}m = \frac{\text{true } T}{\text{trial } T} \cdot v_{32}n = \frac{275.14}{240.00} \cdot (15) = 17.19$$

$$v_1m_1 = \frac{\text{true } T'}{\text{trial } T'} \cdot (v_1n_1) = \frac{250.27}{240.00} \cdot (15) = 15.63$$

A line ($k\ k_1$) was then located which had the same relation to the arch ring that $m\ m_1$ had to the equilibrium polygon. To locate $k\ k_1$ a line was drawn from O to O_1, (centers of the springing line) and the distances $y_1\ y_2\ y_3$ ---, were found, from O O_1 to $a_1\ a_2$ ---. As the arch ring is symetrical it was neccessary to find mean value of the y's for only one half the arch. Letting $x_1\ x_2\ x_3$ ---, be the horizontal distances to the points $y_1\ y_2\ y_3$ ---, (ka.y) was determined for the entire arch. Ordinates above the line $k\ k_1$ were considered positive, and those below as negative. (See Table VI, page 15.)

The next step was to find Σ(mb.y), the ordinates measured above $m\ m_1$ being regarded as positive, and those below as negative. This work is shown in Table VII, page 16.

It is a principle of the equilibrium polygon that if the ordinates are altered in a given ratio, the pole distance is altered in the inverse ratio. The ordinates were altered in the ratio of Σ(ka.y) to Σ(mb.y), and the pole distance, was altered by the inverse of this ratio. The value of the true thrust H was

$$H = \frac{1090.91}{905.04} \times 75000 = 90375 \text{ lb.}$$

To locate the true pole a line was drawn parallel to the closing line of the trial equilibrium polygon, from the trial pole until it intersected the load line. Then a horizontal line was

drawn parallel to the closing line of the trial equilibrium polygon, from the trial pole until it intersected the load line. Then a horizontal line was drawn to the right from this point. The true pole lies on this line at a distance corresponding to 90375 lb. to the right of the load line.

The true equilibrium polygon was then drawn starting at the center of the arch ring, parallel to the rays of the new force polygon. The points where this equilibrium polygon cuts the forces P_1 P_2 P_3--, are designated by c_1 c_2 c_3---.

To find the stresses due to direct stress and bending moment caused by the dead and live loads, it was necessary to resolve the thrust H into two components, tangential and normal to the arch ring at the point at which the stress was to be found. Thus at a_1,

T = Tangential component = $124125 \cdot \cos 7^\circ 10' = 123132$ lb.

and the

N = Normal component = $124125 \cdot \sin 7^\circ 10' = 15518$ lb.

The normal component represents the shear, which amounts to 14 lb. per sq. in. The bending moment is found by multiplying the thrust H by the distance (a. c.) designated as t. Thus at a,

$M = Ht = H(a.c.)$

$H = 90375$ lb. $(a.c.) = 1.1$ ft.

$M = 90375 \; 1.1 = 99412$ ft. lb.

The stress on the concrete is due to the resultant of the direct stress and bending stress and was found at a, as follows:

$$\text{Stress} = \frac{T}{A} \pm \frac{Mv_1}{I}$$

$d_1 = 7.35$ ft. hence $A = 7.35$ sq.ft.

$$\text{Stress} = \frac{123132}{7.35} \pm \frac{(99412)(3.675)}{33.09}$$

T = Tangential thrust
A = Area of arch ring at a_1
M = moment in ft.-lb.
v_1 = distance of extreme fibre from neutral axis.
I = moment of inertia, of section at a_1.

$= 9190 \pm 11040$

$= +\ 30230$ lb. sq.ft. = 210 lb.per sq. in. compression at extrados.

$+\ 8150$ lb. sq.ft. = 60 lb. per sq. in. compression at intrados.

The stresses were found at a_{11}, at the same crown, at a_{24} and a_{32}, in the same manner. (See Table VIII, page 17.)

To find the temperature stresses , the normal temperature was assumed to be 70° F., and that the variation which would occur was assumed to be from 70° to 0° F. This variation of 70°F. is equal to 22 1/9° C. The horizontal thrust (H) was first found by the formulae:-

$$H = \frac{E\ l\ e'\ \theta}{2(\Sigma y^2 - e\Sigma y)} \cdot \frac{I}{S}$$

where E_1 = modulus of elasticity of concrete = 191,600,000 lb. per sq. ft.

l = length of neutral axis between abutments. = 104 ft.

e'= expansion for unit length and 1°C. change of temperature = 0.000012.

θ = total change in temperature = 22 1/9° C.

$2(\Sigma y^2 - e\Sigma y) = 905.04$ from Table VI, page 15.

$I = 1/12\ d^3 = (1/12)(4.5)^3 = 7.594$ for S_1, (S_1 = 2ft.)

Substituting in the above formulae:-

$$H = \frac{(191{,}600{,}000)(104)(0.000012)(22\ 1/9)}{905.04} \cdot \frac{7.594}{2}$$

or, H = 18387 lb.

To find the unit stresses due to this thrust it was first necessary to resolve H, and find its tangential components.

Thus at a_1 T =(18387)(cos 51 40') = 11404 lb.

The bending moment is equal to H.a_1 K_1, and (a_1K) was equal to 14 feet. M = (11404)(14) = 159656 ft.lb. The unit temperature

stress was then found from the above by substituting in the formulae:-

$$\text{Stress} = \frac{T}{d_1} \pm \frac{M\,v_1}{I}$$

$$= \frac{11404}{7.35} \pm \frac{(159656)(3.675)}{33.09}$$

$$= 1551 \pm 17430 = \begin{matrix} +18980 \text{ lb. per sq. ft.} \\ -15880 \text{ lb. per sq. ft.} \end{matrix}$$

= 132 lb. per sq. in. compression at intrados

110 " " " " tension at extrados.

The temperature stress at a_{11}, the crown, a_{24} and a_{32} were found in the same manner. (See Table VIII, page 17.)

CONCLUSION.

From Table VIII , page 17, it is seen that thecombined stresses due to loads and temperature variations, are within the limits set for concrete, in common practice, and, moreover, are not so small as to show a waste of material in construction. The arch is,therefore, a well designed structure.

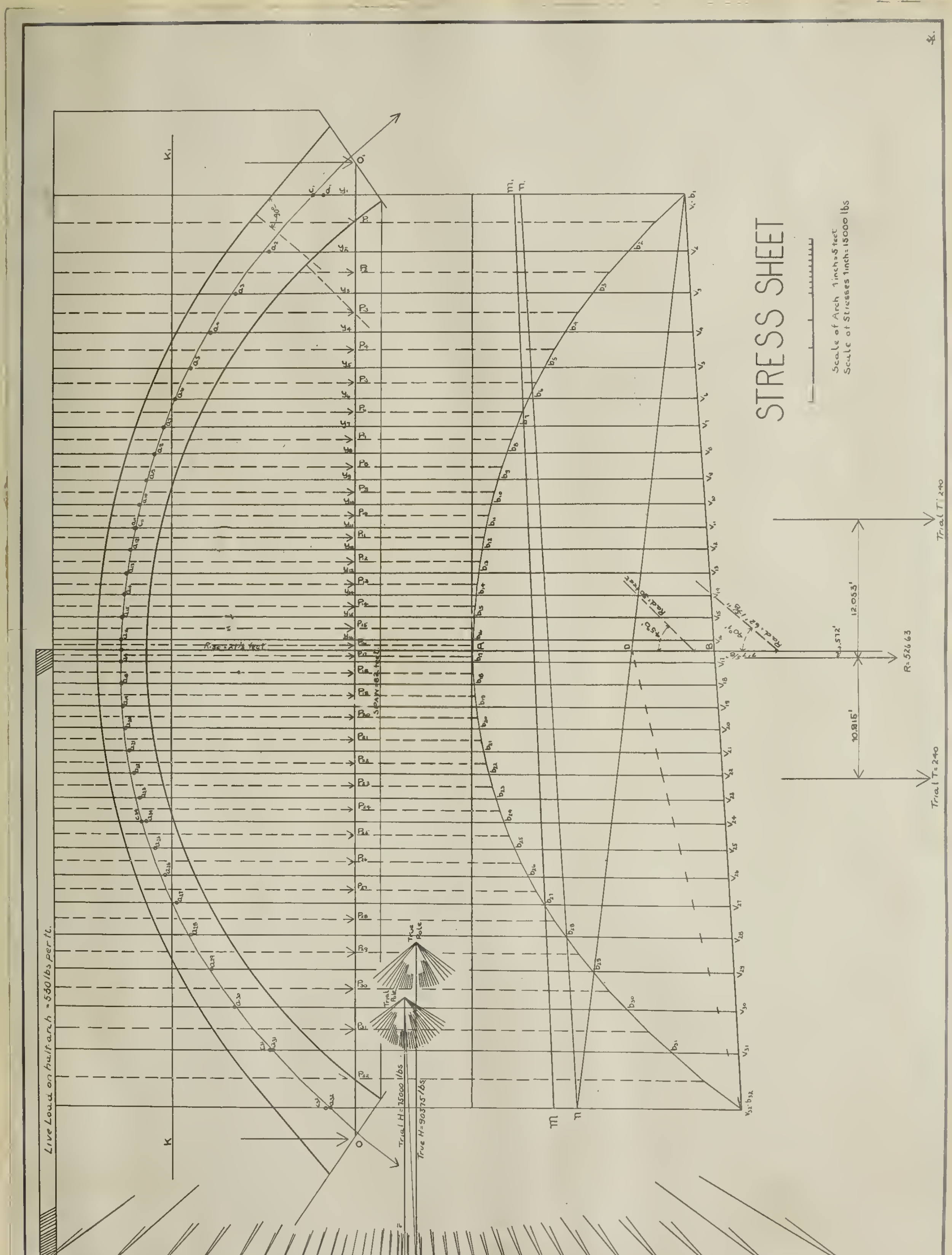
STRESS SHEET
Scale of Arch 1 inch = 5 feet
Scale of Stresses 1 inch = 15000 lbs
Live Load on half arch = 550 lbs per ft.
Trial H = 75000 lbs
True H = 90375 lbs
True Pole
Trial Pole
R = 52663
12.053'
10.815'
Trial T = 240
Trial T = 240

TABLE I.

Depths of Arch Ring.

Distance in feet from crown (L)	Depth of arch ring (d) in feet	L	d	L	d
1	4.50	22	5.08	43	6.69
2	4.50	23	5.12	44	6.82
3	4.50	24	5.15	45	6.98
4	4.51	25	5.23	46	7.18
5	4.53	26	5.30	47	7.35
6	4.54	27	5.35	48	7.55
7	4.55	28	5.41	49	7.78
8	4.56	29	5.50	50	8.02
9	4.57	30	5.57	51	8.30
10	4.59	31	5.62	52	8.58
11	4.61	32	5.70	53	8.89
12	4.65	33	5.78	54	9.20
13	4.69	34	5.85	55	9.60
14	4.72	35	5.93	56	9.92
15	4.75	36	6.00	57	10.30
16	4.80	37	6.09	58	10.60
17	4.82	38	6.18	59	10.15
18	4.88	39	6.26	60	11.60
19	4.92	40	6.35	61	12.05
20	4.98	41	6.44	62	12.60
21	5.02	42	6.53		

TABLE II.

Divisions of Arch Ring.

S	(l) at end of (S)	(L) at middle of (S)	Corresponding (D).
2.00	2.00	1.00	4.50
2.00	4.00	3.00	4.50
2.04	6.04	5.02	4.53
2.08	8.12	7.08	4.55
2.09	10.19	9.17	4.57
2.17	12.36	11.30	4.63
2.25	14.61	13.51	4.68
2.39	17.00	15.83	4.78
2.58	19.58	18.32	4.90
2.77	22.35	21.00	5.08
2.99	25.34	23.88	5.15
3.35	28.69	27.06	5.35
3.85	32.54	30.67	5.60
4.50	37.00	34.55	5.90
5.60	42.64	39.90	6.30
8.84	51.48	47.12	7.39

TABLE III.

Loads on Arch Ring.

Load No.(P)	Dead Load in lb.	Live Load in lb.	Live Dead Loads in lb.
1	14210	2230	16440
2	11100	2070	13170
3	8890	1910	10800
4	6580	1640	8220
5	5880	1564	7444
6	5110	1325	6435
7	4260	1325	5585
8	3840	1280	5120
9	3390	1160	4550
10	2920	1150	4070
11	2800	1120	3920
12	2710	1120	3830
13	2625	1110	3735
14	2605	1110	3705
15	2562	1100	3662
16	1254	445	1699
17	1254		1254
18	2562	0	2562
19	2605	0	2605
20	2625	0	2625
21	2710	0	2710
22	2800	0	2800
23	2920	0	2920
24	3390	0	3390
25	3840	0	3840
26	4260	0	4260
27	5110	0	5110
28	5880	0	5880
29	6580	0	6580
30	8890	0	8890
31	11100	0	11100
32	14210	0	14210

TABLE IV. PART I.

Lengths of Ordinates (Vb) and Distances (Z).

Length of Ordinates in feet.				Distances (Z) in feet.	
v_1b_1		$v_{17}b_{17}$	22.10	Z_2	36.56
v_2b_2	5.60	$v_{18}b_{18}$	22.10	Z_3	32.72
v_3b_3	9.05	$v_{19}b_{19}$	22.05	Z_4	29.16
v_4b_4	11.75	$v_{20}b_{20}$	21.90	Z_5	25.94
v_5b_5	13.91	$v_{21}b_{21}$	21.63	Z_6	23.14
v_6b_6	15.48	$v_{22}b_{22}$	21.31	Z_7	20.55
v_7b_7	16.73	$v_{23}b_{23}$	20.81	Z_8	18.07
v_8b_8	17.71	$v_{24}b_{24}$	20.19	Z_9	15.63
v_9b_9	18.72	$v_{25}b_{25}$	19.32	Z_{10}	13.47
$v_{10}b_{10}$	19.45	$v_{26}b_{26}$	18.29	Z_{11}	11.32
$v_{11}b_{11}$	20.10	$v_{27}b_{27}$	16.95	Z_{12}	9.23
$v_{12}b_{12}$	20.60	$v_{28}b_{28}$	15.20	Z_{13}	7.08
$v_{13}b_{13}$	21.08	$v_{29}b_{29}$	13.01	Z_{14}	5.03
$v_{14}b_{14}$	21.41	$v_{30}b_{30}$	10.12	Z_{15}	3.02
$v_{15}b_{15}$	21.72	$v_{31}b_{31}$	6.21	Z_{16}	1.01
$v_{16}b_{16}$	21.95	$v_{32}b_{32}$			

$\sum(vb) = 526.63$ lb.

TABLE IV. PART II.

Values of (Z.bv)

Ordinates	Differences in feet	Corresponding values of (Z) in feet.	Differences $v_{31}b_{31} - v_2b_2$ etc. X Z'S.
$v_{31}b_{31} - v_2b_2$	.61	36.56	22.30
$v_{30}b_{30} - v_3b_3$	1.07	32.72	35.01
$v_{29}b_{29} - v_4b_4$	1.26	29.16	36.74
$v_{28}b_{28} - v_5v_5$	1.29	25.04	33.46
$v_{27}b_{27} - v_6b_6$	1.47	33.18	33.97
$v_{26}b_{26} - v_7b_7$	1.56	20.55	32.06
$v_{25}b_{25} - v_8b_8$	1.67	18.07	30.18
$v_{24}b_{24} - v_9b_9$	1.47	15.63	22.98
$v_{23}b_{23} - v_{10}b_{10}$	1.36	13.47	18.32
$v_{22}b_{22} - v_{11}b_{11}$	1.21	11.32	13.70
$v_{21}b_{21} - v_{12}b_{12}$	1.03	9.23	9.51
$v_{20}b_{20} - v_{13}b_{13}$	.82	7.08	5.82
$v_{19}b_{19} - v_{14}b_{14}$	.64	5.03	3.22
$v_{18}b_{18} - v_{15}b_{15}$	.38	3.02	1.15
$v_{17}b_{17} - v_{16}b_{16}$	.15	1.01	1.52

$\Sigma(vb.z) = 299.94$ ft.-lb. $R = \dfrac{299.94}{526.63} = 0.569$ ft. to left of AB.

TABLE V.

Ordinates vn.

Difference of ordinates in feet		Distances (Z) in feet		Ordinates X distances in ft. lb.
$v_{32}n - v_1n$	15.00	Z_1 =	41.85	627.75
$v_{31}n - v_2n$	13.09	Z_2	36.56	478.75
$v_{30}n - v_3n$	11.74	Z_3	32.72	384.13
$v_{29}n - v_4n$	10.42	Z_4	29.16	303.85
$v_{28}n - v_5n$	9.38	Z_5	25.94	243.32
$v_{27}n - v_6n$	8.31	Z_6	23.11	192.04
$v_{26}n - v_7n$	7.39	Z_7	20.55	151.86
$v_{25}n - v_8n$	6.45	Z_8	18.07	116.55
$v_{24}n - v_9n$	5.58	Z_9	15.63	87.22
$v_{23}n - v_{10}n$	4.80	Z_{10}	13.47	64.66
$v_{22}n - v_{11}n$	4.05	Z_{11}	11.32	45.84
$v_{21}n - v_{12}n$	3.30	Z_{12}	9.23	30.46
$v_{20}n - v_{13}n$	2.54	Z_{13}	7.08	17.98
$v_{19}n - v_{14}n$	1.69	Z_{14}	5.03	8.50
$v_{18}n - v_{15}n$	1.05	Z_{15}	3.02	3.17
$v_{17}n - v_{16}n$	0.38	Z_{16}	1.01	.38

(Sum of ordinates in triangle nn_1v_1)(Z) equals 2756.28 ft.-lb.
(Sum of ordinates in triangle nv_1v_{32}) (Z) " 2756.28 ft.-lb.
Sum of ordinates in each triangle equals 240.0 $\overline{)11.484}$
(Distance from T to AB)=(Distance from T' to AB)=(2756.28)-(240)=

TABLE VI.

Values of (y)

Ordinates (y)	Length of ordinates (y) in ft.	(y^2)
y_1	2.83	8.01
y_2	7.87	61.94
y_3	10.90	118.81
y_4	13.35	178.81
y_5	15.10	228.01
y_6	16.55	273.90
y_7	17.55	308.00
y_8	18.52	342.99
y_9	19.30	372.49
y_{10}	19.88	395.24
y_{11}	20.35	414.12
y_{12}	20.72	429.32
y_{13}	21.05	443.10
y_{14}	21.28	452.84
y_{15}	21.42	457.81
y_{16}	21.50	462.25

$\sum_0^{L/2} (y) = 268.17$ $\qquad$ $\sum_0^{L/2} (y^2) = 4947.05$

$e = \frac{\sum_0^{L/2} (y)}{16} = 16.76$ ft.

$\sum$ (Ka.y) for the entire arch $= 2(\Sigma y^2 - e\Sigma y) = 905.04$ lb.

TABLE VII.

(Σ mb.y) Values of mb.y

Sum of (mb's)	Value in lb.		Value of (y)		Values (y.mb)	
mb_{16} and mb_{17}	+ 5.52 + 5.61	=	+ 11.13	21.50	239.30	
mb_{15} " mb_{18}	+ 5.32 + 5.67	=	+ 10.99	21.42	235.41	
mb_{14} " mb_{19}	+ 5.10 + 5.58	=	+ 10.68	21.28	227.27	
mb_{13} " mb_{20}	+ 4.75 + 5.35	=	+ 10.10	21.05	212.61	
mb_{12} " mb_{21}	+ 4.33 + 5.08	=	+ 9.41	20.72	194.96	
mb_{11} " mb_{22}	+ 3.85 + 4.70	=	+ 8.55	20.35	173.99	
mb_{10} " mb_{23}	+ 3.24 + 4.13	=	+ 7.37	19.88	146.52	
mb_{9} " mb_{24}	+ 2.58 + 3.48	=	+ 6.06	19.30	116.96	
mb_{8} " mb_{25}	+ 1.74 + 2.58	=	+ 4.32	18.52	80.00	
mb_{7} " mb_{26}	+ 0.70 + 1.49	=	+ 2.19	17.55	38.43	
mb_{6} " mb_{27}	- 0.50 + 0.10	=	- 0.48	16.55		-7.94
mb_{5} " mb_{28}	- 1.98 - 1.70	=	- 3.68	15.10		-55.67
mb_{4} " mb_{29}	- 4.05 - 3.88	=	- 7.93	13.35		-105.87
mb_{3} " mb_{30}	- 6.71 - 6.85	=	- 13.56	10.90		-147.80
mb_{2} " mb_{31}	-10.08 -10.82	=	- 20.90	7.87		-164.48
mb_{1} " mb_{32}	-15.63- 17.19	=	- 32.82	2.83		-92.78

1665.45-574.54

Σ mb.y equals 1665.45 ~~equals~~ 574.54 =1090.91 lb.

TABLE VIII.

Point on Arch.	Total stresses due to loads.		Temperature stresses lb. per sq. in.		Resultant stresses due to both loads and temperature changes.	
	Intrados	Extrados	Intrados	Extrados	Intrados	Extrados.
a_1	+ 60	+210	+132	-110	+192	+100
a_{11}	+179	+100	- 93	+148	+ 86	+248
Crown	+139	+139	-175	+205	- 36	+344
a_{24}	+194	+ 78	- 56	+108	+138	+184
a_{32}	+157	+128	+132	-110	+289	+238

+ designates compression.

— designates tension.